

# Physics 604, Lecture Notes Supplement

## Electrostatic Units

Prof. Charles E. Hyde-Wright  
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### I. COULOMB'S LAW

Coulomb's Law of the force between two isolated charges  $q_1$  and  $q_2$  is:

$$|\mathbf{F}| = k_1 \frac{q_1 q_2}{r_{1,2}^2} \quad (1)$$

where  $r_{1,2}$  is the distance between charges  $q_1$  and  $q_2$ . Instead of thinking of the charges as abstract "point-charges", it is probably better to think of  $q_1$  and  $q_2$  as charges each separately confined to a volume of radius  $a$ . Coulomb's law is the limiting value of the force between the two charges as  $a/r_{1,2} \rightarrow 0$ .

### II. UNITS

What about the constant  $k_1$  in Eq. 1? The existence of  $k_1$  and the fact that it is a scalar quantity, is in part the empirical deduction of Mr. Coulomb. The numerical value of  $k_1$  depends entirely up to our choice of definition of units for all the other quantities: Force, Distance, and Charge.

There are many systems of units in use in physics, engineering, and general use (although the general public does not much use electromagnetic units). There is an excellent appendix in Jackson on units. It is imperative that you read it, and read it again when we introduce magnetic units later in the semester. In this course, we will use three systems of units:

- SI: *Système Internationale*, or MKS-Ampère.
- Gaussian, or cgs-Gauss, or cgs-electrostatic
- Heaviside-Lorentz

You will perhaps be relieved that we will not use any system based on "English" (now American) mechanical units of feet and pounds.

#### A. MKSA

You are probably most familiar with the MKSA system of units. All physical quantities are measured either as multiples of the fundamental units of meters, kilograms, seconds, and amperes (electric current), or as multiples of derived units. Some examples of derived units are

- Force: 1 Newton (N) =  $1kgm/s^2$ ;
- Energy: 1 Joule (J) =  $1kgm^2/s^2$ ;
- Charge: 1 Coulomb (C) = 1 Ampere sec = 1 A s;
- Electrostatic Potential: 1 Volt (V) =  $1J/C$ ;
- Magnetic Field: 1 Tesla (T) =  $1Vs/m^2$ .
- *et cetera*.

The Ampere is defined by the magnetic forces generated by currents (see Chapter XXX). Having defined the Ampere, the value of  $k_1$  must be determined by empirical measurement. Also, the value of the elementary charge of the proton and electron must also be measured:

$$k_1 = 8.99 \cdot 10^9 Nm^2/C^2 \quad (2)$$

$$q_{\text{proton}} = -q_{\text{electron}} = 1.602177 \cdot 10^{-19}C \quad (3)$$

Rather than  $k_1$ , in the MKSA system, we more often use the *permittivity of free space*:

$$\epsilon_0 = 1/(4\pi k_1) = 8.85 \cdot 10^{-12} \text{Farad/m} \quad (4)$$

In the MKSA system, Coulomb and Gauss' laws have the forms:

$$F_{1,2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{1,2}^2} \quad (5)$$

$$\int \mathbf{E} \cdot d\mathbf{S} = Q/\epsilon_0 \quad (6)$$

### III. GAUSSIAN UNITS

In the Gaussian, cgs, or electrostatic system of units, we define the unit of charge of 1.0 electrostatic units (esu) such that the constant  $k_1$  is **defined** as:

$$k_1 = 1.0 \text{dynecm}^2/(\text{esu})^2 \quad (7)$$

In gaussian units, Coulomb and Gauss' laws are:

$$F_{1,2} = \frac{q_1 q_2}{r_{1,2}^2} \quad (8)$$

$$\int \mathbf{E} \cdot d\mathbf{S} = 4\pi Q \quad (9)$$

$$q_{\text{proton}} = -q_{\text{electron}} = 4.803\,206 \cdot 10^{-10} \text{esu} \quad (10)$$

The elementary charge is: Remember:

- 1 dyne = 1 gm cm/s<sup>2</sup> = 10<sup>-5</sup> N;
- 1 erg = 1 dyne·cm = 10<sup>-7</sup> J;

In Comparing the units of charge