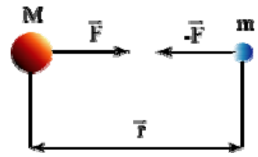
The motion of the Planets

See www.nasa.gov for more information

Gravitational force

Gravitational force is one of the four fundamental forces



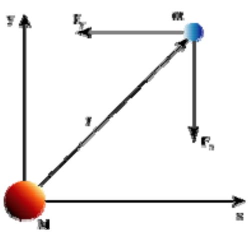
$$\vec{F} = -G\frac{mM}{r^2}\hat{r} = -G\frac{mM}{r^3}\vec{r}$$

$$G = 6.67\cdot 10^{-11}\frac{m^3}{kg\cdot s^2}$$

Simple case: motion in a central field (m<<M)

Equation of motion

$$m\frac{d^2\vec{r}}{dt^2} = -G\frac{mM}{r^3}\vec{r}$$



Properties of the motion in the central field

Total energy is conserved

$$E = \frac{1}{2}mv^2 - G\frac{mM}{r}$$

Angular momentum is conserved

$$L_z = (\vec{r}\times m\vec{v})_z = m(xv_y - yv_x)$$

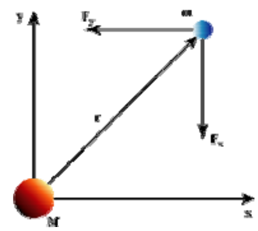
Motion in a central field (m<<M) (cont.)

Equation of motion in Cartesian coordinates

$$\frac{d^2x}{dt^2} = -G\frac{M}{r^3}x$$

$$\frac{d^2y}{dt^2} = -G\frac{M}{r^3}y$$

$$r^2 = x^2 + y^2$$

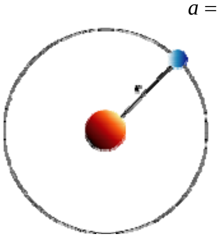


Initial value ODE problem:

equations + initial conditions (position and velocity)

Motion in a central field (circular orbit)

For circular orbits $a = \frac{v^2}{r}$ $m\frac{v^2}{r} = G\frac{mM}{r^2}$



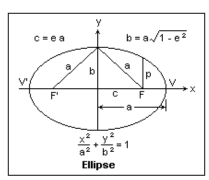
then …

$$v = \left(G\frac{M}{r}\right)^{1/2}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{(GM)^{1/2}}$$

Motion in a central field (elliptical orbit)

For circular orbits



Properties:

For any point on the elliptical orbit

distance from F + distance form F' = const

It is common to specify elliptical orbits by the semi major axis *a* and eccentricity *e*

$$e = \sqrt{1-\frac{b^2}{a^2}}$$

The eccentricity of the Earth orbit is about 0.0167, but this number changes with time.

Solar system

Astronomical units

distance: 1 AU = 1.496*10^{11} m

Time: 3.15*10^{7} s

In this units:

distance Earth to Sun is about 1 AU

The period of the Earth is about 1 year

and

$$GM = \frac{4\pi^2}{T^2} r^3 = 4\pi^2 \cdot AU^3 / years^2$$

Planet	t (years)	d(AU)	e	mass
Mercury	0.241	0.387	0.206	0.055
Venus	0.615	0.723	0.007	0.815
Earth	1.000	1.000	0.017	1.000
Mars	1.880	1.523	0.093	0.107
Jupiter	11.86	5.202	0.048	317.8
Saturn	29.5	9.539	0.054	95.16
Uranus	84.0	19.18	0.047	14.37
Neptune	165	30.06	0.009	17.15
Pluto	*248*	*39.44*	*0.249*	*0.002*

Solar system

More data on Solar system

http://hyperphysics.phy-astr.gsu.edu/hbase/solar/soldata2.html

http://www.solstation.com/stars/sol-sum.htm

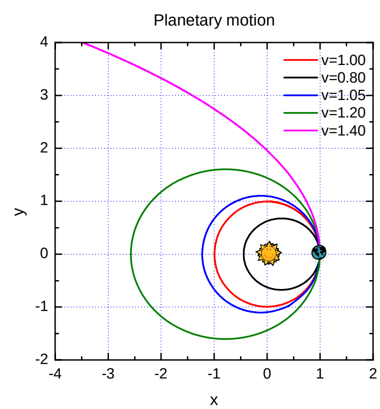
Planetary motion

Problem

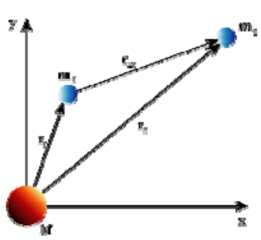
Write a program to simulate the Earth's orbit around the Sum

- Considering the special case of a circular orbit $v_0=(GM/r)^{1/2}$.
- Calculate motion with initial velocities: v = 0.8, 1.1, 1.2, 1.3, 1.4 v_0.
- Check conservation of energy with time.
- Check conservation of angular momentum.
- Find the numerical value of the period for each v.
- Find the numerical value of eccentricity for each v

Example



A mini solar system



Equations of motion

$$m_1 \frac{d^2\vec{r}_1}{dt^2} = -G\frac{Mm_1}{r_1^3}\vec{r}_1 + G\frac{m_1 m_2}{r_{21}^3}\vec{r}_{21}$$

$$m_2 \frac{d^2\vec{r}_2}{dt^2} = -G\frac{Mm_2}{r_2^3}\vec{r}_2 - G\frac{m_1 m_2}{r_{21}^3}\vec{r}_{21}$$

where

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

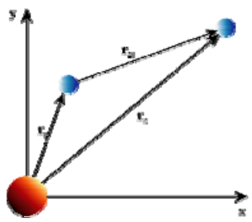
Planetary motion 2

Problem

Write a program to simulate the motion of two planets (for example Earth and Venus) around the Sun

- Initial conditions: two circular orbits when the planets do not mutually interact ($v_0=(GM/r_i)^{1/2}$.)
- Study the effect of mutual interaction
- Are there total energy and angular momentum conserved?
- Are the energy and angular momentum of planet 1 conserved?

The classic helium atom



Equations of motion

$$\frac{d^2\vec{r}_1}{dt^2} = -\frac{2}{r_1^3}\vec{r}_1 + \frac{1}{r_{12}^3}(\vec{r}_1 - \vec{r}_2)$$

$$\frac{d^2\vec{r}_2}{dt^2} = -\frac{2}{r_2^3}\vec{r}_2 + \frac{1}{r_{21}^3}(\vec{r}_2 - \vec{r}_1)$$

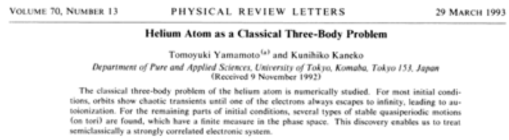
The classical helium atom

Problem

Write a program to simulate the motion of two electrons in 2D model of the classical helium atom

Choose units: electron charge = 1, electron mass = 1

- Study the effect of mutual interaction (i.e. study motion with and without electron-electron interaction



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Helium Atom as a Classical Three-Body Problem

Tomoyuki Yamamoto[a] and Kunihiko Kaneko

Department of Pure and Applied Sciences, University of Tokyo, Komaba, Tokyo 153, Japan

(Received 9 November 1992)

The classical three-body problem of the helium atom is numerically studied. For most initial conditions, orbits show chaotic transients until one of the electrons always escapes to infinity, leading to autoionization. For the remaining parts of initial conditions, several types of stable quasiperiodic motions (on tori) are found, which have a finite measure in the phase space. This discovery enables us to treat semiclassically a strongly correlated electronic system.