Macroscopic Observables of Matter

- So far: treated extended objects either as
  - Mass points… (momentum, force laws)
  - ...or as rigid objects (rotation, torque,…) *)
- BUT: even a cube of 1 mm side length contains about 10^{19} molecules/atoms!
- We could describe each one of them as a mass point (or their nuclei and electrons)
  -> impossibly complicated
- …or try to find "macroscopic" observables that describe the bulk properties of matter (including internal degrees of freedom) without getting into details => **Thermodynamics**

*) Except last lecture where we began to study extended media with ∞ degrees of freedom

Density

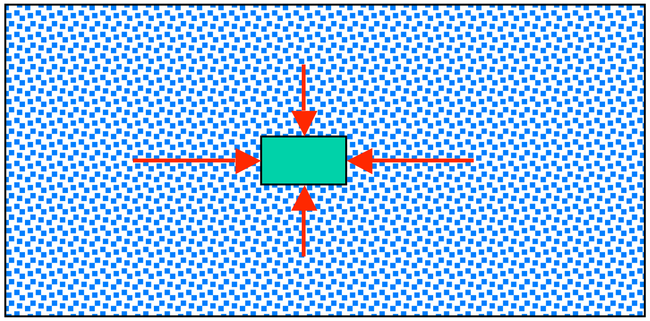
- The number of molecules (atoms) included in some volume (small part of an object or fluid) is roughly proportional to that volume
  - The mass m_V of the matter contained in the volume V is approximately proportional to V
  - The ratio $\rho = m_V / V$ approaches a finite value as V is made smaller and smaller
  - ρ is called the (local) mass **density** of the material; unit kg/m^3
- Notes:
  - Mathematical abstraction; only valid if V doesn't become **too** small (compared to size of molecule)
  - Typical mass densities are around 1 - 10,000 kg/m^3:
    ρ_{Air} = 1.2 kg/m^3 , ρ_{Water} = 1000 kg/m^3 ,
    ρ_{Al} = 2700 kg/m^3 , $\rho_{Platinum}$ = 21400 kg/m^3
    Examples: air in room, 1 liter of gold

Molecular Densities

- Definition: One mol of the Carbon isotope ^{12}C weighs exactly 0.012 kg (12 g).
- Empirical finding: One mol contains about $N_A = 6.022137 \cdot 10^{23}$ Carbon atoms (Avogadro/Loschmidt number)
- General definition: A mol of **any** substance contains that same number (N_A) of molecules of that substance
- Mass of one mol
  $= 0.012$ kg · (Molecular Mass/12)
  $= 1$ g · (Molecular Mass)
- Examples: Water molecules in 1 mm^3 , Air molecules in the room

Pressure

- Mechanical definition:
  Perpendicular force per unit area
- Same mathematical abstraction: if force is spread out evenly, total force on an area A $F_{perp}(A)$ will be proportional to A :
  => ratio $p = F_{perp}(A)/A$ will become constant as A is made smaller and smaller
- => Pressure p ; unit N/m^2 = Pa (Pascal)
- Inside a fluid: consider tiny cube, forces on each face are equal
  => pressure is a scalar (just one number)

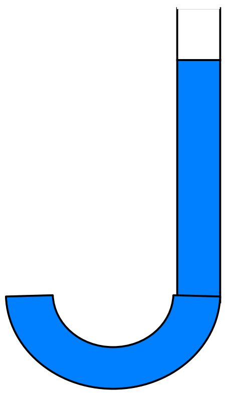


Examples for pressure

- Average atmospheric pressure:
  101,325 Pa => force on 1 cm^2: 10.1 N
  (1 kg Weight) (huge!)
  - Lots of funny units: bar, mbar, atm, psi, mm of Mercury (torr), inches of Mercury…
    1 atm = 101 kPa = 1013 mbar = 14.7 psi
    = 760 Torr = 29.9 inHg
- Hydrostatic pressure: Weight of tiny volume $W = mg = \rho V g = \rho g A \Delta y$
  => $p_{\text{Bottom}} - p_{\text{Top}} = \rho g \Delta y$
  => Pressure at any height h above reference level: $p(h) = p(0) - \rho g h$
- Pressure is the same at equal heights throughout a fluid (Pascal's Law) (Hydraulics)
- Archimedes' Principle; floating and buoyancy

Consequences of hydrostatic pressure & Pascal's Law

- Hydraulics: Convert small force on small surface into large force on large surface
- Air pressure decreases with height (12 Pa/m)
- Water pressure increases with depth (a lot) (9800 Pa/m)
- Use to measure relative ("gauge") pressure



- Use to measure absolute pressure
- Other gauges: compression of reference volume, expansion of gas,...

Buoyant Force

- Object partially submerged in liquid: Pressure on bottom is $P_{\text{bottom}} = \rho_{\text{Liquid}}\, g\, \Delta y$ => $F_{\text{buoy}} = \rho_{\text{Liquid}}\, g\, \Delta y\, A$ = Weight of displaced liquid
- Fully submerged object: $F_{\text{buoy}} = \rho_{\text{Liquid}}\, gh\, A$ => Object will float if density less than that of liquid
- Example: Buoyant force in air (person, helium balloon)
- Example: Buoyant force in water: Ships, icebergs
  - Puzzle 1: Dipping finger in beaker
  - Puzzle 2: Boat with rock in pond