# Work and Energy

- Work: The total **effect** a force **F** has **on** an object (mass point, closed system,…) after some change in its position
  - Given by Force x Displacement (see later)
  - Changes motional (kinetic) state (or internal state, mechanical state,…)
  - Can be positive or negative
  - Analogy: Transfer of money
- Energy: The **ability** to **do** work
  - Property **of** the object (mass point, closed system,…)
  - Changed by work done **on** object
  - Can be exchanged between objects/systems, but **can not** be created or destroyed (energy conservation)
  - Analogy: Net Worth

# Work and Kinetic Energy I

- Prototype example: (1D motion with constant acceleration
  - Average velocity (1):
    $v_{av,x}(0 \ldots t) = {}^{1}/_{2}\,(v_x(t) + v_{x\,0})$
    $= (x(t) - x_0)/(t - 0)$
  - Average acceleration (2):
    $a_{av,x} = (v_x(t) - v_{x\,0})/(t - 0)$
  - l.h.s (2) x r.h.s. (1) = l.h.s (1) x r.h.s. (2) =>
    $a_{av,x}\,(x(t) - x_0)/(t - 0) =$
    ${}^{1}/_{2}\,(v_x(t) + v_{x\,0})(v_x(t) - v_{x\,0})/(t - 0) \Rightarrow$
    $a_{av,x}\,(x(t) - x_0) = {}^{1}/_{2}\,(v_x^2(t) - v_{x\,0}^2)$
  - Multiply with mass m =>

$$m a_{av,x}\,(x(t) - x_0) = {}^{1}/_{2}\,m\,(v_x^2(t) - v_{x\,0}^2)$$

$F_x$ $\Delta x$ K.E. ($t$) K.E. (0)

# Work and Kinetic Energy II

- Dimension: Displacement times Force
  Unit: Nm = J (Joule)     Symbol: $W$

1) Specify a force acting **on** an object
2) Multiply displacement in the direction of the force with that force:
   $\Delta W = F_x \Delta x + F_y \Delta y + F_z \Delta z =$
   $\mathbf{F} \cdot \Delta \mathbf{s} = F_{||} \Delta s = F \Delta s \cos\phi$
3) Add work done due to all (external) forces acting on object:
   $\Delta W = \Sigma \mathbf{F}_\mathrm{i} \cdot \Delta \mathbf{s} = \Sigma \Delta W_i$
4) Set equal to change in kinetic energy:
   $\Delta \mathrm{K.E.} = {}^{\mathrm{m}}/_2\, v_\mathrm{f}^2 - {}^{\mathrm{m}}/_2\, v_\mathrm{i}^2 = \Delta W$

- Example: Pushing car up incline, car rolling down (demo)

# Important Notes

- Previous definition only true for (approximately) constant net force (constant acceleration). General Form:

$$\Delta W = \int_{r(\text{initial})}^{r(\text{final})} \vec{\mathbf{F}} \cdot \mathbf{d}\vec{\mathbf{s}} = \frac{m}{2} v_{\text{final}}{}^{2} - \frac{m}{2} v_{\text{initial}}{}^{2}$$

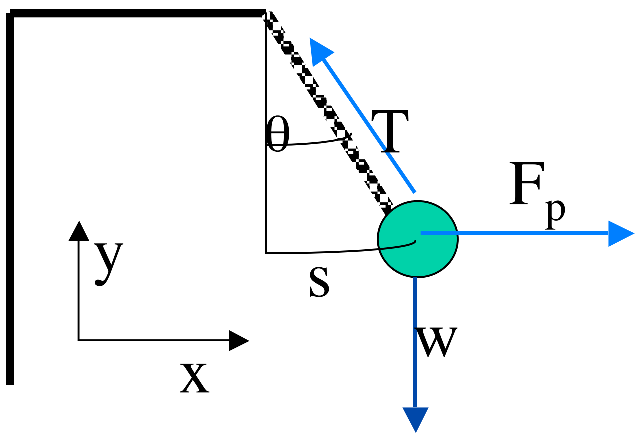
  (Proof)
- Example: Compression of a spring
- Work done on object can be positive (object increases kinetic energy) or negative (object decreases kinetic energy). Example: Vertical throw
- Often (but not always), positive work done on one object is accompanied by negative work done on another (Newton's 3$^{rd}$ law) Example: Baseball catcher

# More Important Notes

- Even if $\Sigma \mathbf{F}_i \neq 0$, Work done can be zero:
  - No displacement: Holding a book, pushing against a wall
  - Direction of displacement perpendicular to Force: Moving sideways (constant height) in gravity field, circular motion (constant speed)
  - Normal forces and static friction **never** do work, but tension can and kinetic friction can, too (always negative).
- Kinetic energy is a scalar quantity:
  K.E. = $^{m}/_{2}\, v^2 = W\,(0 \rightarrow v)$, always positive
  - Independent of direction (circular motion at constant speed doesn't change kinetic energy)
  - Depends on system of reference
- **Everything** I said **only** valid in Inertial System of Reference

# Work on a pendulum - an Example

- Slowly pushing a pendulum bob sideways:




$$\sum F_x = F_p - T\sin\theta$$
$$\sum F_y = -mg + T\cos\theta$$
$$T = mg/\cos\theta$$
$$F_p = mg\tan\theta$$
$$s = \theta R; \quad ds_{\parallel} = R\cos\theta\, d\theta$$

$W(F_p) = m\,g\,R\,(1 - \cos\theta)$ ; $W_{tot} = 0$

- Letting go:
  $W(\text{grav}) = m\,g\,R\,(1 - \cos\theta) = W_{tot}$
  $^{m}/_{2}\, v^2\,(\text{bottom}) = m\,g\,R\,(1 - \cos\theta)$

# Power

- Work done per unit time:
  $P = \Delta W / \Delta t$ (average) ->
  $dW/dt$ (instantaneous)
- Unit: Watt = Joule/s, kW (kiloWatt) ->
  new unit for energy&work: Ws, kWh…
- $\Delta W = \mathbf{F} \cdot \Delta \mathbf{s} \Rightarrow P = \mathbf{F} \cdot \Delta \mathbf{s} / \Delta t = \mathbf{F} \cdot \mathbf{v}$
- Example:
  1000 kg Car accelerating from 0 to 20m/s in 5 s => $F$ = 4000 N,
  $v_{ave}$ = 10 m/s => $P_{ave}$ = 40 kW = 53 hp .
- Different way to calculate:
  K.E. (final) = 200,000 J = $\Delta W$ in 5 s

# Analogy - again

| | |
|---|---|
| • Object (Mass Point, System) | • Person |
| • External Source of Force | • External Source of Money (Bank,...) |
| • Positive Work done **on** object by 1 force (motion ǁ to force) | • Cash given **to** person by external source |
| • Negative Work done **on** object by 1 force (motion opposite to force) | • Cash spent **by** person |
| • Total Work done | • Net Cash flow |
| • Kinetic energy of object | • Total cash in person's pocket |
| • Power | • Cash flow / time |
| • K.E. = $^{m}/_{2}\, v^{2}$ | |
| • $\Delta W = \int \vec{\mathbf{F}} \cdot \mathbf{d}\vec{\mathbf{s}}$ | |