Putting it all together…

- Newton's 3 laws: a = F/m, action = reaction
- Linear motion: position, velocity, acceleration, momentum
- Circular motion: angular velocity, angular momentum, centripetal acceleration
- Energy (kinetic, potential,…)
- OUR FIRST REAL FORCE LAW: $F = G\, m\, M \,/\, r^2$

- **=> 3D motion in a gravitational field**

First case: near-surface projectiles

- Can ignore the curvature of Earth's surface - pretend it's flat
- Can ignore variations of the strength and direction of the gravitational force - pretend it's always mg straight down, with $g = 9.8$ m/s^2 fixed.
- Need to distinguish 2 directions: "Horizontal" = x (along Earth's surface) and "Vertical" = y (up and down).
- Position described by **two** functions $x(t)$, $y(t)$.
  Velocity has also **two** components: $v_x(t)$ and $v_y(t)$.
  Acceleration is **only** in $-y$ – direction.

Main point

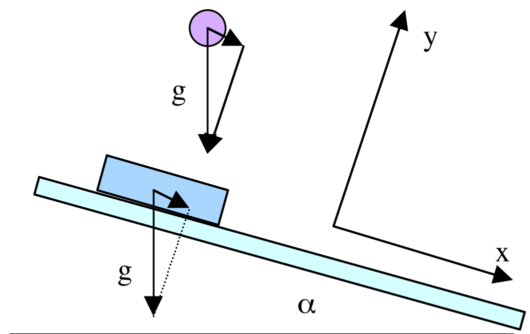
- Motion in horizontal direction is completely decoupled from motion in vertical direction as long as we can ignore air resistance (break down all vectors - forces, accelerations, velocities into their *x*- and *y*-components).
- Horizontal equations:
  $a_x = 0$; $v_x = \text{const.}$; $x(t) = x_0 + v_x t$
- Vertical equations:
  $a_y = -g$; $v_y(t) = v_{y0} - gt$
  $y(t) = y_0 + v_{y0} t - {}^1/_2\, gt^2$
- Example: stone dropped from tree *vs*. thrown stone; sack dropped from airplane.

More examples

- Horizontal launch: $|\mathbf{v}_0| = 9$ m/s ; $x_0 = 0$ m ; $y_0 = 150$ m
  => t_{impact} ? $\mathbf{v}_{impact}$? x_{impact} ?
- Launch at an angle:
  - Time in flight (largest for straight up)
  - Distance traveled (equal for 30^o and 60^o, maximum for 45^o) (PHeT)
- Ball launched from moving cart
- Ball launched from accelerated cart
  - Pulling force
  - Inclined track

WHY?

- Choose x-axis along ramp

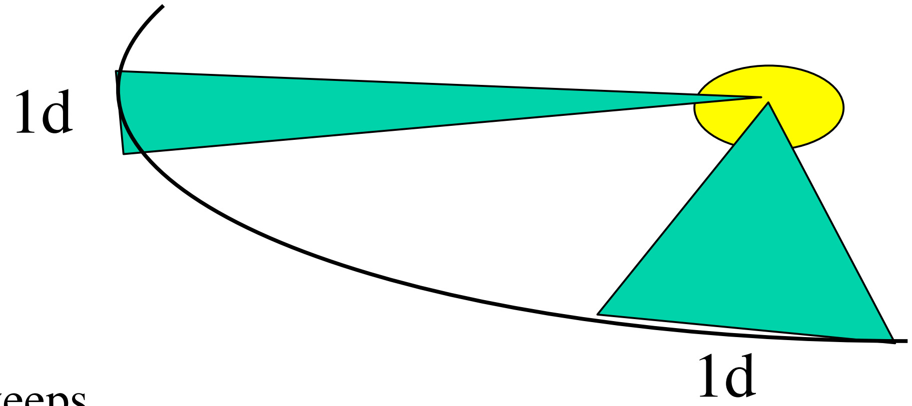


- x-components of acceleration and velocity are equal
- Motion in y is independent

Second case: Satellite Motion

- What happens if you throw horizontally, but faster and faster?
  - Earth is "curving away underneath flightpath"
  - Direction of gravitational force changes over time
- => if speed is fast enough, you get circular motion instead of parabolic one, and no impact at all => Satellite!
- Minimum required speed at surface of Earth: $g = v^2/R \Rightarrow v = \sqrt{gR} = \sqrt{9.8\text{m/s}^2 \times 6380{,}000\text{m}} = 7900\text{m/s}$
- Acceleration is 3600 times smaller where the moon is -> need only 1/60 of the speed to stay aloft!
- Geostationary satellites: $v/R = 2\pi/86400\text{s}$

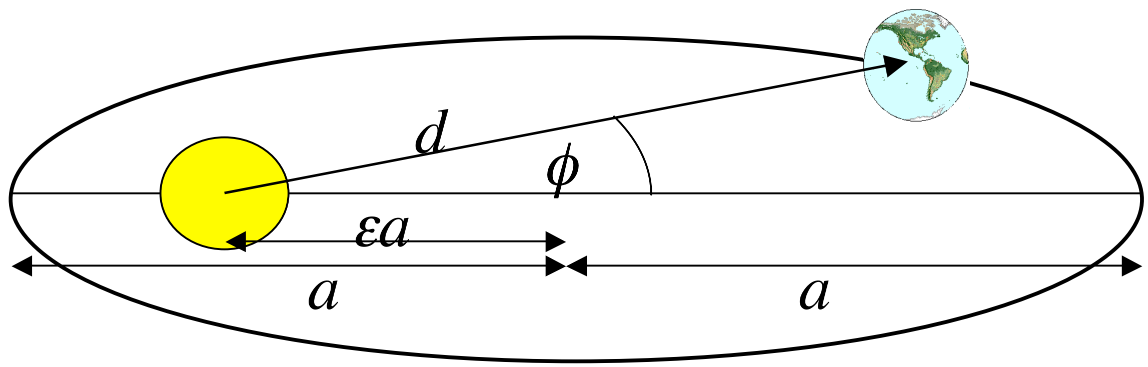
More general: Motion of planets and satellites



- Kepler's Laws:
  - Orbits of satellites (moons, planets,…) are elliptical
  - A line from the gravitating body (sun, planet,…) to the satellite sweeps out equal areas in equal times. (Conservation of angular momentum)
  - The period T of an orbit is proportional to $a^{3/2}$ (a = major half axis of ellipse) and $1/\sqrt{M}$.
- Circular orbit at distance R from central object with constant angular velocity ω
  - Centripetal force: $F_{\mathrm{rad}} = m R \omega^2$
  - Gravitational force: $F_{\mathrm{grav}} = GmM/R^2$
  - => $\omega^2 = 4\pi^2/T^2 = G M/R^3$
    => $T = 2\pi (R^3/GM)^{1/2}$
  - Example: Satellite vs. Moon - Example: Miniature solar system

Motion of planets and Satellites (cont'd)

- Elliptical orbit



  - Gravitating body (sun, planet,…) located in one focal point of ellipse
  - Total " length" of ellipse is 2 a ("major half axis")
  - "Eccentricity" ε = distance from focal point to center/a
  - Major half axis a replaces R in 3rd Law:
    $T \propto a^{3/2}$
  - Conservation of angular momentum L requires $\omega \propto 1/d^2$ => "areas" law
  - => Kepler's Laws derived from Newton's Laws

Gravitational Potential Energy

- Potential energy change = -Work done by gravity
- Close to Earth: $F = mg \Rightarrow W = -mg\Delta h \Rightarrow U = mgh$
- Moving further away, force is not constant but decreases, so energy increases less slowly: $\Delta U = GmM / r^2 \, \Delta r$
- Note: only motion away from or towards source of gravitational attraction changes potential energy
- General form for 2 masses m, M at a distance D:
  $U_{\text{grav}} = -GmM / D$
- Note: always negative! (We chose the reference point where $U = 0$ infinitely far away in this case)
- Note: It takes a FINITE amount of energy to escape Earth (the sun, moon, …) FOREVER!

Examples

- Two steel balls from before:
  (M = 10 kg, m = 5 kg, d = 0.1 m apart)
  - Initially $U_{grav} = -GmM/D = -3.34 \cdot 10^{-8}$ J
  - Touch at 1cm distance: $U_{grav} = -3.34 \cdot 10^{-7}$ J
  - Total kinetic energy increases by $3.0 \cdot 10^{-7}$ J
  - Mass m will have twice the speed as mass M (momentum conservation)
    => K.E. = $m/2\, v^2 + M/2\, v^2/4 = 3/4\, mv^2$
    => v = $8.9 \cdot 10^{-5}$ m/s (< 0.1 mm/s) (**not** a constant acceleration!)
- Escape velocity from Earth:
  - First $U_{grav} = -GmM_{Earth}/R_{Earth}$, K.E. = $m/2\, v^2$
  - Finally, $U_{grav} = 0$ and K.E. = 0
  - Energy conserved
    => $GmM_{Earth}/R_{Earth} = m/2\, v^2$
    => $v^2 = 2GM_{Earth}/R_{Earth} = 2gR_{Earth}$ => v = 11,200 m/s

Extreme Example: Black holes

- Make radius R smaller and smaller while keeping M constant:
  - Collapsing stars (supernovae)
  - Large initial density fluctuation in the Universe ("primordial black holes", quasars); center of most (all?) galaxies
  - High energy collisions?
- $v^2 = 2GM/R$ becomes larger and larger, until it exceeds the speed of light c^2 => NOTHING can escape once it is closer than R to the center
- All matter crushed to "infinitely small" center inside black hole
- Huge tidal forces because of $1/R^3$
- Complete description requires Einstein's GENERAL theory of relativity (curvature of space-time)