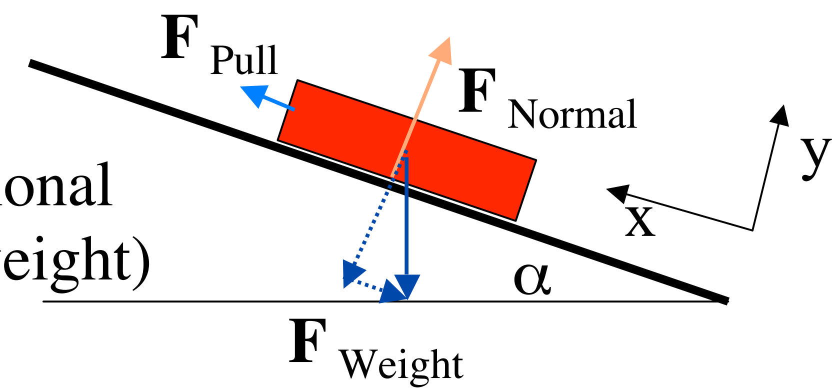
Potential Energy

- So far: Considered all forces equal, calculate work done by net force only => Change in kinetic energy.
  - Analogy: Pure Cash Economy
- But: Some forces seem to be able to "store" the work for you (when they do negative work) and "give back" the **same** amount (when they do positive work).
  - Analogy: Bank Account. You pay money in (ending up with less cash) - the money is stored for you - you can withdraw it again (get cash back)
- These forces are called "conservative" (they conserve your work/money for you)

Potential Energy - Example

- Car moving up ramp: Weight does negative work
  $\Delta W(\text{weight}) = -mg\Delta h$
- Depends only on initial and final position
- Can be retrieved as positive work on the way back down
- Two ways to describe it:
  1) No net work done on car on way up
  2) Pulling force does positive work that is stored as gravitational potential energy $\Delta U = -\Delta W(\text{weight})$



Total Mechanical Energy

- Dimension: Same as Work
  Unit: Nm = J (Joule)   Symbol: $E = K.E. + U$

1) Specify all **external** *) forces acting **on** a system
2) Multiply displacement in the direction of the net external force with that force:
   $\Delta W_{ext} = F \, \Delta s \cos\phi$
3) Set equal to change in total energy:
   $\Delta E = {}^{m}/_{2} v_f^2 - {}^{m}/_{2} v_i^2 + \Delta U = \Delta W_{ext}$
   $\Delta U = -W_{int}$

*) We consider all non-conservative forces as external, plus all forces that we don't want to include in the system.

Example: Gravitational Potential Energy *)

- I. Motion in vertical (y-) direction only:

  $$\Delta U = -W_{grav} = mg\Delta y$$

- External force: Lift mass m from height y_i to height y_f (without increasing velocity) => Work gets stored as gravitational potential energy $\Delta U = mg\,(y_f - y_i) = mg\,\Delta y$
- Free fall (no external force): Total energy conserved, change in kinetic energy compensated by change in potential energy $\Delta K.E. = {}^{m}/_{2}\, v^2 = -mg\,\Delta y$
- Example: Throw baseball up with 20 m/s (accelerate over 0.5m). Maximum height? Force needed?

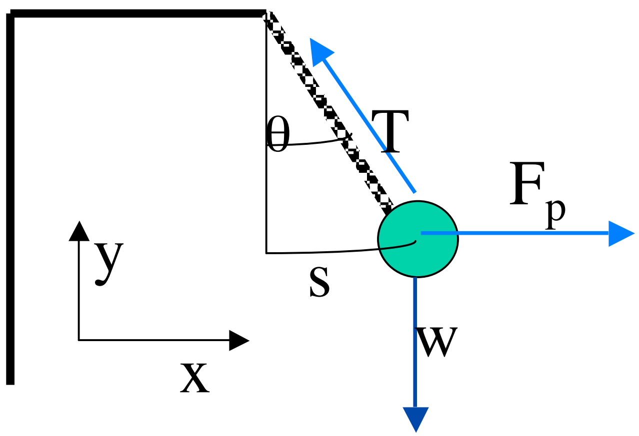
*) Here the system consists of Earth plus object!

Gravitational Potential Energy II

- Important point: Potential energy has **no** absolute zero (like kinetic energy does - in a given reference frame!).
- Depends on choice of **reference point**: **You decide** where you want U to be 0.
- Call that point $h = 0$.
- Define potential energy as $U = mgh$.
- => Total energy $E = {}^{m}/_{2}\, v^2 + mgh$.
- Choice arbitrary - other choice means constant offset in definition of U and E .
- No **observable** depends on that choice! All that counts are differences ΔU.
  BUT: you must specify reference point when quoting E and U !

Work on a pendulum - an Example

- Slowly pushing a pendulum bob sideways:



1) Tension does no work (perpendicular to motion)
2) Pushing force does work - equal to potential energy stored by gravitational force
3) After letting go, gravitational potential energy gets converted to kinetic energy at the bottom

Pendulum cont'd.

- On the way up: Net work done by external force (pushing) increases total energy ($K.E. + U_{grav}$) stored in the system
- On the way down: No other (non-grav.) force => Total Energy conserved ($\Delta E = 0$) $\Delta E = \Delta\text{K.E.} + \Delta U = 0$ => $\Delta\text{K.E.} = {}^{m}/_{2}\, v^2 = -\Delta U = -mg\Delta h$
- Important points:
  - Minus sign: **negative** work done by conservative force **increases** the potential energy due to that force (putting cash into account)
  - Potential energy is stored in **system** (pendulum and gravitation, wallet and bank account)
  - Total energy changes through work done by forces **external** to system (push, cash influx)
  - System must be "leakproof" (conserve work): return to initial condition -> same potential energy (conservative forces)

Elastic Potential Energy

- So far: Considered system object - gravitational field;
  Potential energy = - gravitational internal work (inside system)
- Now: Consider system cart - spring; Potential energy = - internal work done by spring
  - Call x = 0 unstretched position of spring
  - Force exerted by spring $F_x = -kx$
  - Work done by spring $\Delta W = - k/2\,(x_f^2 - x_i^2)$
  - Potential energy stored in spring-cart system:
    $U = -\Delta W = k/2\,(x_f^2 - x_0^2)$ where x_0 is the point where we declare U to be $U = 0$.
  - $x_0 = 0 \Rightarrow U = k/2\, x^2$ (convenient, **not** unique)
  - Note: $U > 0$ stretched **and** compressed

Elastic Potential Energy cont'd.

- No other (non-elastic) force =>
  Total Energy conserved ($\Delta E = 0$)
  $\Delta E = \Delta \text{K.E.} + \Delta U = 0 \Rightarrow \Delta \frac{m}{2} v^2 = -\Delta U$
  Example: Oscillation
- Non-elastic force present =>
  $\Delta E = \Delta \text{K.E.} + \Delta U = \Delta W \Rightarrow$
  $\Delta \frac{m}{2} v^2 = -\Delta U + \Delta W$
- Elastic and gravitational force present =>
  $E = \text{K.E.} + U_{el} + U_{grav} =$
  $\frac{m}{2} v^2 + k/2\, x^2 + mgh$.
  (several bank accounts)
- Note: Elastic forces are conservative because work done only depends on initial and final position.

Total Mechanical Energy - Final Version

1) Specify all forces acting on an object
2) Separate out all **conservative** forces (Work done depends only on initial and final position). Incorporate them into the system of the object as potential energy U
3) Add all **external** forces acting **on** the system (= all other forces), call the result "net (external) force".
4) Multiply displacement in the direction of the net force with that force: $\Delta W_{ext} = \mathbf{F} \cdot \Delta \mathbf{s} = F \, \Delta s \cos\phi$
5) Set equal to change in total energy: $\Delta E = {}^{m}/_{2}\, v_f^2 - {}^{m}/_{2}\, v_i^2 + \Delta U = \Delta W_{ext}$
6) $\Delta U = -\Delta W_{int}$; $\Delta\, {}^{m}/_{2}\, v^2 = -\Delta U + \Delta W_{ext}$

Examples: Pumpkin falling on spring-loaded platform (without and with air resistance); bungee jump

Other types of Energy

1) Electromagnetic energy (see later in the semester).
   Examples: Charged capacitors (electrostatic energy), current-carrying coils (magnetic energy), …
2) Chemical energy (really a special kind of electromagnetic energy).
   Examples: Batteries, fuel, …
3) Sound, light, nuclear,…
4) Internal energy (heat, pressure,…) -see next semester (PHYS102)

Important Points

- Energy concept is useful:
  - Calculate change in velocity without knowing Force as F(t)
  - Understand levers, hydraulic systems, mechanical advantage…
- Energy concept is fundamental: Energy is conserved!
  - Kinetic energy (always positive - "cash")
  - Gravitational energy (can be + or -; "credit card account")
  - Elastic potential energy (always positive - "savings")
  - Chemical, electrical, sound, light, …. Energy
  - Heat energy (less useful - "cash hidden in underwear")
- Efficiency = fraction of useful work/total energy transferred