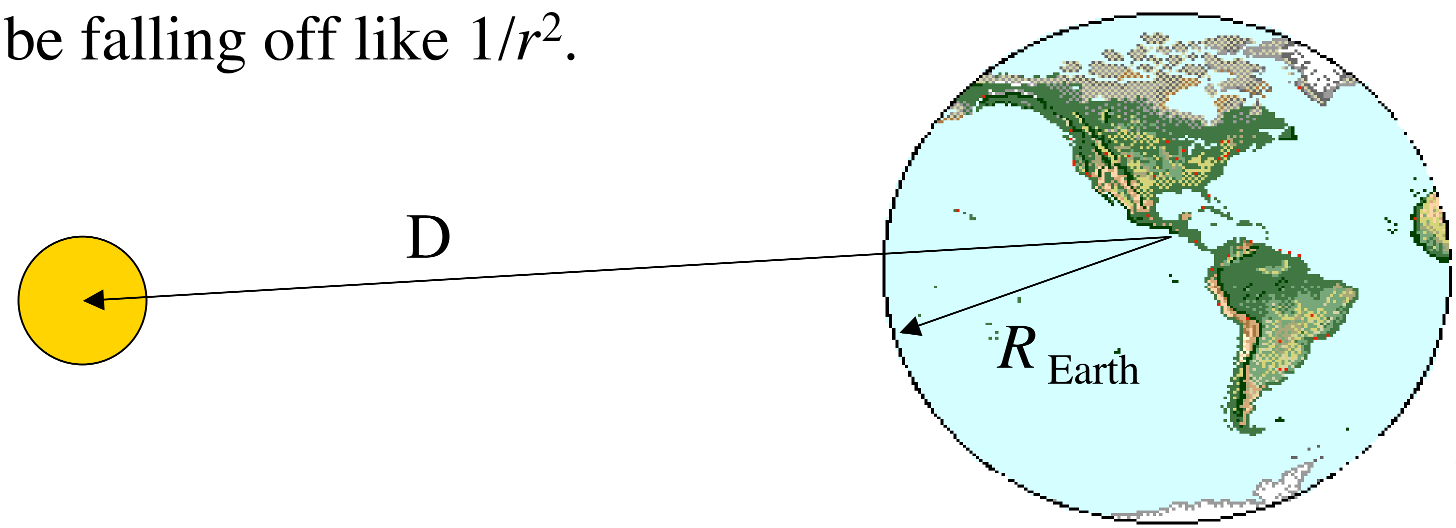
Gravitation

- Objects on the surface of Earth fall down with acceleration $a_{rad} = g = 9.81$ m/s^2 ; Earth's radius is $R_{Earth} = 6380$ km.
- Moon circles the Earth once every 27.3 d = $2.36 \cdot 10^6$ s => $\omega = 2.66 \cdot 10^{-6}$/s . Moon is $D = 384{,}000$ km away => $a_{rad} = D\omega^2 = 0.00272$ m/s^2 (3600 times smaller). D is 60 times bigger than R_{Earth} ! => Gravitational force must be falling off like $1/r^2$.



Newton's Law of Gravitation

- All masses are accelerated with the same acceleration at the same distance from Earth => $F \propto m$ (since $a = F/m$)
- Newton's 3rd law => Earth is attracted to any mass with force proportional to M => $F \propto m\,M$
- The force is proportional to the distance squared: $F \propto 1/r^2$
- Need proportionality constant: G =>
  $F = G\,m\,M/r^2$
- Universal constant => Universal force law for any two bodies with masses m , M at distance r !
- Measure G using torsion balance => $G = 6.7 \cdot 10^{-11}$ N m^2/kg^2
- Plug in numbers: on the surface of earth,
  $g = F/m = G\,M/R_{\text{Earth}}^2$ => $M = 5.97 \cdot 10^{24}$ kg

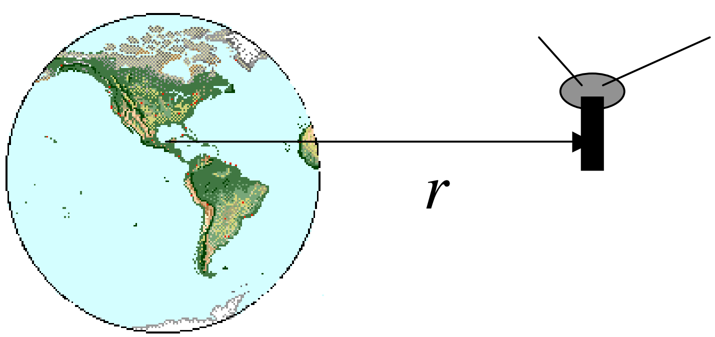
Important points

$$\vec{F}(\text{on } m_2 \text{ at } \vec{r}_2 \text{ due to } m_1 \text{ at } \vec{r}_1) = -G\frac{m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^2}\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

- Universal $1/r^2$ force law - describes not only gravity, but also electromagnetism …
- Valid not only for point masses, but for spherical extended masses as well (measure *r* from the center)
- G can be measured with torsion balance (but it's hard because it is so small) -> Value: $6.673 \cdot 10^{-11}$ N m^2/kg^2
- Force always acts **on** center of gravity of an object (roughly equal to center of mass)
- Force always points **to** center of gravity of attracting mass (along distance vector)
- Mass *m* exerts equal and opposite force on mass *M* as that exerted by *M* on *m* .
- Superposition: Gravitational forces add

Example I

- Project "Lagrange Point"
  Put a satellite between Sun and Earth where the net gravitational force is zero
  - Satellite can be stationary (looking at Earth)
- Where to put?
  - $M_{\text{Earth}} = 6{\cdot}10^{24}$ kg , $M_{\text{Sun}} = 2{\cdot}10^{30}$ kg , $D_{\text{E-S}} = 1.5{\cdot}10^{11}$ m .
  - Require $GmM_{\text{Earth}}/r^2 = GmM_{\text{Sun}}/(D_{\text{E-S}} - r)^2$
    => $(D_{\text{E-S}} - r) = 577\, r$ => $r = D_{\text{E-S}}/578$ = 259,000 km.



Example II

- Two steel balls floating in space (initially at rest). Masses $M = 10$ kg, $m = 5$ kg, $d = 0.1$ m apart.
  - Initial gravitational attraction: $F = G\, m\, M / d^2 = 3.34 \cdot 10^{-7}$ N (on each)
  - Initial acceleration: $3.34 \cdot 10^{-8}$ m/s^2 for M, $6.68 \cdot 10^{-8}$ m/s^2 for m.
  - After 60 s, first one has moved 0.06 mm. Second has moved 0.12 mm.
  - Center of mass remains at same point (1/3 of the way from first to second mass).

Tidal forces

- Because gravitational forces vary like $1/r^2$, they can be different on different parts of the same object.
- Example: Gravitational force exerted by moon on Earth: Largest at point closest to moon, less large on center of Earth, least on opposite side.
- Variation of force over an object of diameter D: $\Delta F/m = \mathrm{D}\cdot\mathbf{2}GM/r^{\mathbf{3}}$
- Tend to elongate objects along connecting line
- Examples: Oceans on Earth (and atmosphere, Earth itself); Moon; black holes