

Classical Mechanics - Problem Set 2

Problem 1)

Solve Problem 18., page 67, in Goldstein.

Problem 2)

A mass point (mass m) is moving in the x - y plane. There are no external forces present, but the mass is tethered to the origin ($x = y = 0$) by a string of fixed length R . This constraint can be expressed by requiring that the function $g(x,y) = 0$, where $g(x,y) = x^2 + y^2 - R^2$. Using Lagrangian multipliers, write down the Euler-Lagrange equations of motion for the variables x and y . A solution can be found by making the ansatz $x = R\cos\phi$ and $y = R\sin\phi$ for some parameter $\phi(t)$ (of course we know what that is, but the point is that we're not supposed to start out with ϕ as generalized coordinate, but only use it at this point). Obviously, this solution "automatically" fulfills the constraint. Show that you can now solve the set of two equations for x and y both for $\phi(t)$ and for the Lagrangian multiplier. From your solution, extract the x - and y -components of the force of constraint exerted by the string on the mass point.

Problem 3)

Two masses m_1 and m_2 move under their mutual gravitational attraction in a uniform external gravitational field whose acceleration is g . Choose as coordinates the Cartesian coordinates X, Y, Z of the center of mass vector \mathbf{R} (taking Z in the direction of g) and the spherical coordinates r, θ and ϕ that define the relative vector \mathbf{r} from m_1 and m_2 (see HW Problem Set 1 for the definition of \mathbf{r}). Write down the Lagrangian in terms of these 6 coordinates. Calculate all 6 generalized momenta. Which of these 6 momenta is conserved? Write down the 6 Euler-Lagrange equations of motion.

Problem 4)

A body in a gravitational field (acceleration g) is released from a height h and after a time T it strikes the ground, $T = \sqrt{2h/g}$. The equation for the distance of fall s after a time t is of course $s = 1/2 gt^2$, but assume someone (I) proposes as an alternative the more general form $s = 1/2 gt^2 + A \sin(\pi t/T)$, where A is an arbitrary constant of dimension length. Note that this gives the same answer for $t = 0$ and $t = T$, meaning the total path h between release and ground contact is traveled in the same time T . By calculating the Lagrange function for this proposed trajectory (with s as the single generalized coordinate), find out which value of A yields a minimum for the integral in Hamilton's principle:

$$\int_{t=0}^{t=T} L(s, \dot{s}, t) dt$$

. (Note that you have to express L as a function of t first before calculating the integral).