

Classical Mechanics - Problem Set 6

Problem 1)

A rectangular block of mass $M = 6\text{kg}$ height $H = 0.2\text{m}$ and square cross section $W \times W = 0.1\text{m} \times 0.1\text{m}$ is rotating freely in space. At some time t , the block is rotating with an instantaneous angular velocity of $\omega = 10\pi \text{ rad/s}$ around an axis that makes an angle θ of 30 degrees with the z' -axis, which is the axis fixed along the H-direction of the block.

- a) What is the magnitude and direction (i.e., its angle with respect to the z' -axis) of the angular momentum \mathbf{L} ?
- b) What is the magnitude of the kinetic energy T ?
- c) How fast will the angular velocity vector ω precess around the z' -axis (as seen from the rotating body system)?

Problem 2)

A car wheel with new tire is clamped into a machine which makes the wheel rotate around a constant axis. Assume the wheel has already been balanced to make the center of mass coincide with the axis of rotation. However, the axis is slightly “out of whack”, so that the principal 3-axis of the inertia tensor of the wheel makes a small angle α with the axis of rotation (assume that the other two principal values I_1 and I_2 are the same). In terms of the angular velocity of the rotation ω , what is the necessary torque exerted by the apparatus on the wheel to keep it on axis? If we want to apply this torque by exerting a force on one end of the axis, in which direction does the force have to point? (By measuring this force, the mechanic can determine how to balance the tire so that the axis of rotation and the 3-axis coincide, which is important for obvious reasons).

Problem 3) – only for your amusement (NO CREDIT – but you can use it for your project!)

Solve Problem 20, page 236, in Goldstein.