Classical Mechanics - Problem Set 9

Problem 1)

Solve Goldstein's Problem 28, page 427.

Problem 2)

Solve Goldstein's Problem 30, page 427.

Problem 3)

A long elastic rod of uniform mass density λ can be described by the longitudinal displacement $q(z)$ of each of its parts from its "relaxed" position z along the rod. The resulting tension at some point z is proportional to the local relative elongation, $\delta\ell/\ell$, of the piece of the rod at position z, relative to its relaxed state. Therefore, the tension in the rod at position z is given by $\delta\ell/\ell$ and by Young's modulus: $T(z) = Y\,\delta\ell/\ell$. The potential energy stored in a length dz of the rod is thus $dV=1/2\ Y\,(\delta\ell/\ell)^2(z)\ dz$. In analogy to the example in the lecture (for a long string under tension), write down the Lagrangian density for this situation. Use the extended Euler-Lagrange equations for continuous 1-dimensional systems to derive the possible dynamic modes of the rod. If the rod has finite length Z, what are the harmonic frequencies of the possible standing waves?