Vector Addition

IDENTIFY *the relevant concepts and* SET UP *the problem:*

Decide what your target variable is. It may be the magnitude of the vector sum, the direction, or both.

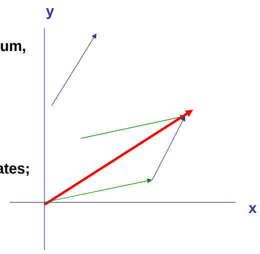
Then draw the individual vectors being summed and the coordinate axes being used.

In your drawing, place the tail of the first vector at the origin of coordinates;

place the tail of the second vector at the head of the first vector;

and so on…

Draw the vector sum $\vec{R}$ from the tail of the first vector to the head of the last vector.



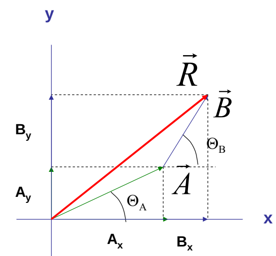
Vector Addition

EXECUTE *the solution as follows:*

1. Find the x- and y-components of each individual vector

record your results in a table.

	x	y
$\vec{A}$	$A\cos\Theta_A$	$A\sin\Theta_A$
$\vec{B}$	$B\cos\Theta_B$	$B\sin\Theta_B$
$\vec{R}$	R_x	R_y



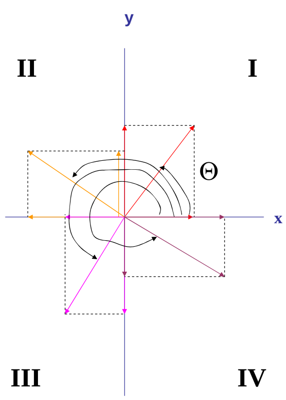
$$R_x = A\cos\Theta_A + B\cos\Theta_B$$
$$R_y = A\sin\Theta_A + B\sin\Theta_B$$

Vector Addition

EXECUTE *the solution as follows:*

Some components may be positive and some may be negative, depending on how the vector is oriented (that is, what quadrant Θ lies in).

	I	II	III	IV
A_x	+	-	-	+
A_y	+	+	-	-



Vector Addition

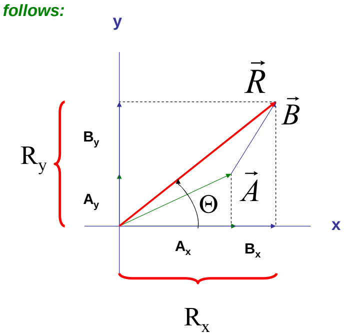
EXECUTE *the solution as follows:*

magnitude:

$$R = \sqrt{R_x^2 + R_y^2}$$

direction:

$$\Theta = a\tan\left(\frac{R_y}{R_x}\right)$$



Problem-Solving Strategy

Magnetic Field Calculations

IDENTIFY *the relevant concepts:* The law of Biot and Savart allows you to calculate the magnetic field due to a current-carrying wire of any shape. The idea is to calculate the field due to a representative current element in the wire, then combine the contributions from all such elements to find the total field.

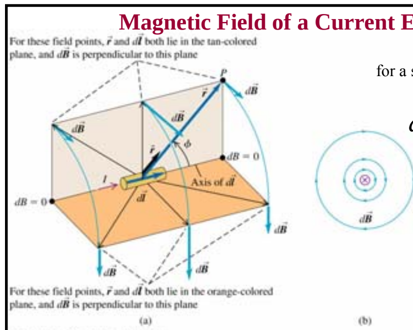
SET UP *the problem using the following steps:*

1. Make a diagram showing a representative current element and the point *P* at which the field is to be determined (the field point).
2. Draw the current element $d\vec{l}$, being careful to ensure that it points in the direction of the current.
3. Draw the unit vector $\hat{r}$. Note that it is always directed *from* the current element (the source point) to the field point *P*.
4. Identify the target variables. Usually they will be the magnitude and direction of the magnetic field $\vec{B}$.

EXECUTE *the solution as follows:*

1. Use Eq. (28.5) or (28.6) to express the magnetic field $d\vec{B}$ at *P* from the representative current element.

Magnetic Field of a Current Element



Biot-Savart Law

for a small element of conductor

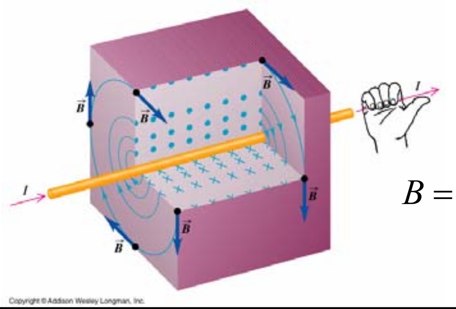
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$

Total Magnetic Field over the entire conductor

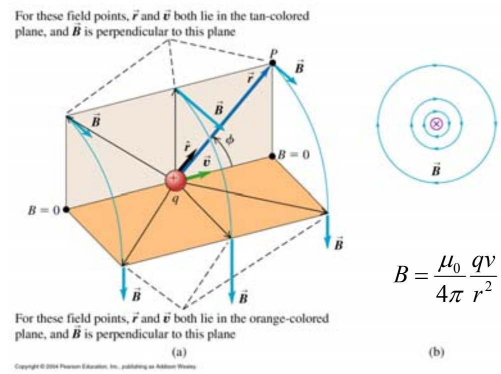
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\hat{r}}{r^2}$$

Magnetic field around a long, straight, current-carrying conductor

The field lines are circles,
with directions determined by the right-hand rule.



$$B = \frac{\mu_0 I}{2\pi r}$$



$$B = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \quad (28.2)$$

(magnetic field of a point charge with constant velocity)

$$B = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \quad \text{(magnetic field of a current element)} \quad (28.6)$$

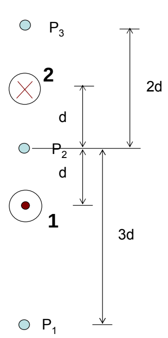
$$B = \frac{\mu_0 I}{2\pi r}$$

2. Add up all the '$d\vec{B}$'s to find the total field at point *P*. In some situations the '$d\vec{B}$'s at point *P* have the same direction for all the current elements; then the magnitude of the total $\vec{B}$ field is the sum of the magnitudes of the '$d\vec{B}$'s. But often the '$d\vec{B}$'s have different directions for different current elements. Then you have to set up a coordinate system and represent each $d\vec{B}$ in terms of its components. The integral for the total $\vec{B}$ is then expressed in terms of an integral for each component.
3. Sometimes you can use the symmetry of the situation to prove that one component of $\vec{B}$ must vanish. Always be alert for ways to use symmetry to simplify the problem.
4. Look for ways to use the principle of superposition of magnetic fields. Later in this chapter we'll determine the fields produced by certain simple conductor shapes; if you encounter a conductor of a complex shape that can be represented as a combination of these simple shapes, you can use superposition to find the field of the complex shape. Examples include a rectangular loop and a semicircle with straight line segments on both sides.

EVALUATE *your answer:* Often your answer will be a mathematical expression for $\vec{B}$ as a function of the position of the field point. Check the answer by examining its behavior in as many limits as you can.

Example 28.4 (modified)

End view of two long, straight, parallel wires perpendicular to the xy-plane, each carrying a current I, but in opposite directions.
a) Find the magnitude and direction of $\vec{B}$ at points P_1, P_2, and P_3. I = 1.0 A; d = 0.1 m.
b) Find the magnitude and direction of $\vec{B}$ at any point on the y axis above wire 2 in terms of the y-coordinate of the point.



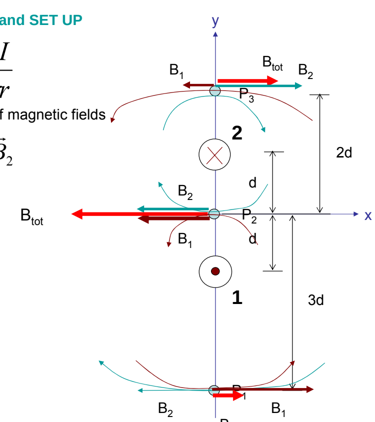
Example 28.4 (modified)

IDENTIFY and SET UP

$$B = \frac{\mu_0 I}{2\pi r}$$

Principle of superposition of magnetic fields

$$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2$$



Example 28.4 (modified)

EXECUTE

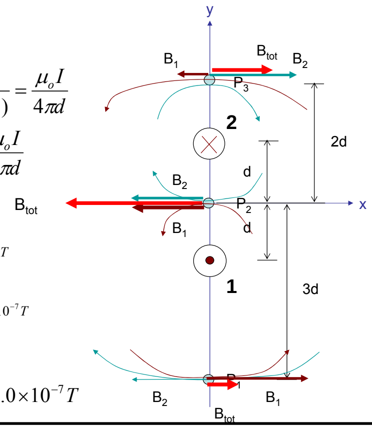
a) $P_1: B_1 = \frac{\mu_o I}{2\pi r_1} = \frac{\mu_o I}{2\pi(2d)} = \frac{\mu_o I}{4\pi d}$

$B_2 = \frac{\mu_o I}{2\pi r_2} = \frac{\mu_o I}{2\pi(4d)} = \frac{\mu_o I}{8\pi d}$

$B_1 = \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)\times(1.0A)}{(4\pi)(0.1m)} = 10^{-6}T$

$B_2 = \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)\times(1.0A)}{(8\pi)(0.1m)} = 5\times10^{-7}T$

$B_{tot} = B_x = B_1 - B_2 = +5.0\times10^{-7}T$



Example 28.4 (modified)

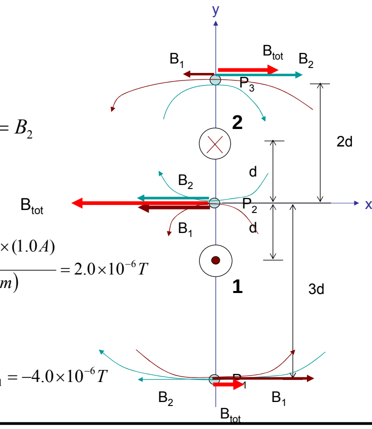
EXECUTE (cont.)

a) (cont.)

$P_2: B_1 = \frac{\mu_o I}{2\pi r_1} = \frac{\mu_o I}{2\pi d} = B_2$

$B_1 = B_2 = \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)\times(1.0A)}{(2\pi)\times(0.1m)} = 2.0\times10^{-6}T$

$B_{total} = B_x = -B_1 - B_2 = -2B_1 = -4.0\times10^{-6}T$



Example 28.4 (modified)

EXECUTE (cont.)

a) (cont.)

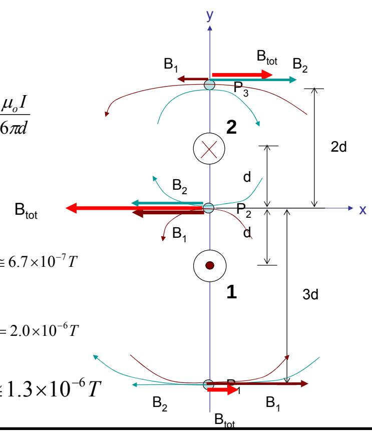
$P_3: B_1 = \frac{\mu_o I}{2\pi r_1} = \frac{\mu_o I}{2\pi(3d)} = \frac{\mu_o I}{6\pi d}$

$B_2 = \frac{\mu_o I}{2\pi r_2} = \frac{\mu_o I}{2\pi d}$

$B_1 = \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)\times(1.0A)}{(6\pi)\times(0.1m)} \cong 6.7\times10^{-7}T$

$B_2 = \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)\times(1.0A)}{(2\pi)\times(0.1m)} = 2.0\times10^{-6}T$

$B_{total} = B_x = B_2 - B_1 \cong 1.3\times10^{-6}T$



Example 28.4 (modified)

EXECUTE (cont.)

b)

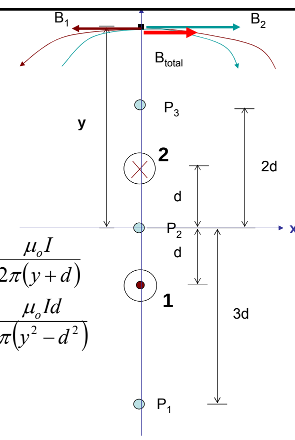
$B_1 = \frac{\mu_o I}{2\pi r_1} = \frac{\mu_o I}{2\pi(y+d)}$

$B_2 = \frac{\mu_o I}{2\pi r_2} = \frac{\mu_o I}{2\pi(y-d)}$

$B_{total} = B_2 - B_1 = \frac{\mu_o I}{2\pi(y-d)} - \frac{\mu_o I}{2\pi(y+d)}$

$B_{total} = \frac{\mu_o I[y+d-(y-d)]}{2\pi(y^2-d^2)} = \frac{\mu_o I d}{\pi(y^2-d^2)}$

$y >> d \therefore B_{total} \to \frac{\mu_o I d}{\pi y^2}$



Example 28.4 (extended)

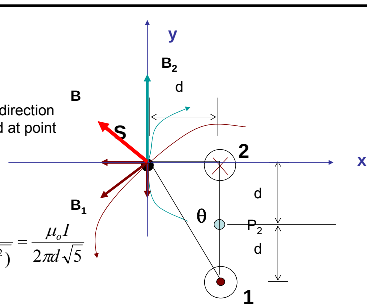
Find the magnitude and direction of the total magnetic field at point S.

$B_1 = \frac{\mu_o I}{2\pi r_1} = \frac{\mu_o I}{2\pi\sqrt{(d^2+4d^2)}} = \frac{\mu_o I}{2\pi d\sqrt{5}}$

$B_2 = \frac{\mu_o I}{2\pi r_1} = \frac{\mu_o I}{2\pi d}$

$B_{1x} = -B_1\cos\theta = -B_1\frac{2d}{d\sqrt{5}} = -\frac{\mu_o I}{2\pi d\sqrt{5}}\times\frac{2}{\sqrt{5}} = -\frac{\mu_o I}{5\pi d}$ $\quad B_{2x} = 0$

$B_{1y} = -B_1\sin\theta = -B_1\frac{d}{d\sqrt{5}} = -\frac{\mu_o I}{10\pi d}$ $\quad B_{2y} = B_2$



Example 28.4 (extended)

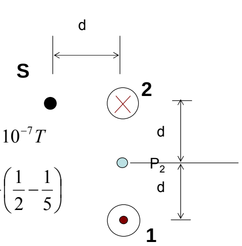
$B_x = B_{1x} + B_{2x} = B_{1x} = -\frac{\mu_o I}{5\pi d} = -8.0\times10^{-7}T$

$B_y = B_{1y} + B_{2y} = -B_1\sin\theta + B_2 = \frac{\mu_o I}{\pi d}\left(\frac{1}{2}-\frac{1}{5}\right)$

$B_y = 0.3\frac{\mu_o I}{\pi d} = 1.2\times10^{-6}T$

$B = \sqrt{B_x^2 + B_y^2} \cong 1.44\times10^{-6}T$

$\varphi = a\tan\left(\frac{B_y}{B_x}\right) = 180^o - 56.3^o = 123.7^o$

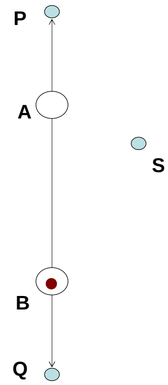


PROBLEM 28.55 (modified)

Consider the diagram:
AB = 1.00m; AS=0.6m BS=0.8m
AP = 0.5m; BQ = 0.5m; I_B=18.0A

(a) What must the magnitude and direction of the current I_A be for the net field at point *P* to be zero?

(b) Then, what are the magnitude and direction of the net field at point *Q*?
(c) Then, what is the magnitude of the net field at point *S*?



PROBLEM 28.55 (modified)

IDENTIFY and SET UP

$$B = \frac{\mu_0 I}{2\pi r}$$

Principle of superposition of magnetic fields

$$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2$$

EXECUTE

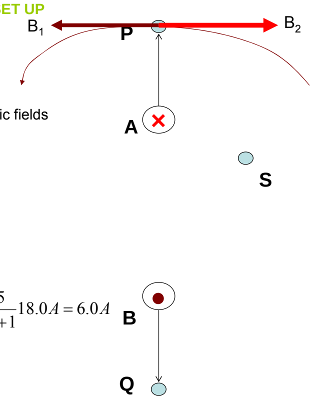
$$a)P : B = 0 \therefore B_2 = -B_1$$

$$\frac{\mu_o I_A}{2\pi r_2} = -\left(\frac{-\mu_o I_B}{2\pi r_1}\right)$$

$$I_A = \frac{r_2}{r_1} I_B = \frac{(\overline{AP})}{(\overline{AP}+\overline{AB})} \times I_B = \frac{0.5}{0.5+1} 18.0A = 6.0A$$

Into the plane

AB = 1.00m; AS=0.6m BS=0.8m
AP = 0.5m; BQ = 0.5m; I_B =2.0A



PROBLEM 28.55 (modified)

EXECUTE

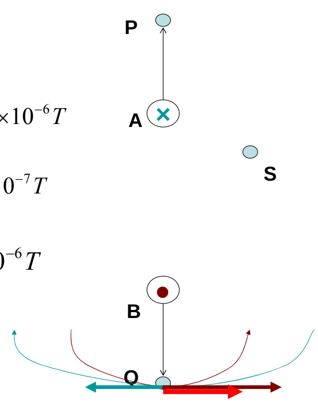
$$b)Q : B_1 = \frac{\mu_o I_B}{2\pi(\overline{BQ})} = 7.2\times10^{-6} T$$

$$B_2 = \frac{\mu_o I_A}{2\pi(\overline{AB}+\overline{BQ})} = 8\times10^{-7} T$$

$$B = B_1 - B_2 = 6.4\times10^{-6} T$$

$$\theta = 0$$

AB = 1.00m; AS=0.6m BS=0.8m
AP = 0.5m; BQ = 0.5m; I_B =2.0A



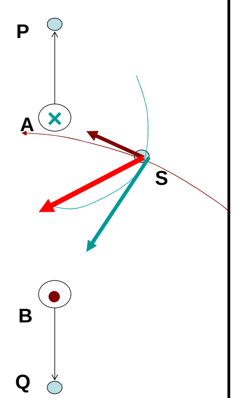
PROBLEM 28.55 (modified)

EXECUTE

$$B_S = \sqrt{B_1^{\,2}+B_2^{\,2}} = \frac{\mu_0}{2\pi}\sqrt{\left(\frac{I_1}{r_1}\right)^2+\left(\frac{I_2}{r_2}\right)^2} =$$

$$= \frac{\mu_0}{2\pi}\sqrt{\left(\frac{6.00\text{ A}}{0.60\text{ m}}\right)^2+\left(\frac{2.00\text{ A}}{0.80\text{ m}}\right)^2} = 2.1\times10^{-6}\text{ T.}$$

AB = 1.00m; AS=0.6m BS=0.8m
AP = 0.5m; BQ = 0.5m; I_B =2.0A



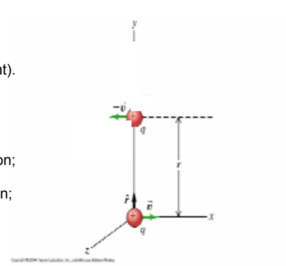
EXAMPLE 28.1 (modified)

Magnetic fields between two moving charges

Two protons move parallel to the x-axis in opposite directions at the same speed v = 1000 km/s (small compared to the speed of light).
At the instant shown (r = 1μm), find the direction and the magnitude of the magnetic fields:

a) At the position of the upper proton;

b) At the position of the lower proton;



EXAMPLE 28.1 (modified)

IDENTIFY and SET UP

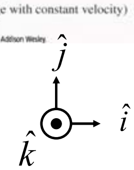
right-hand rule

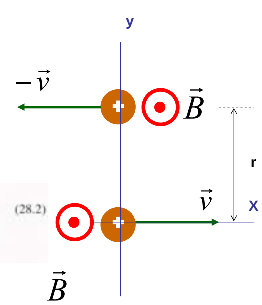
$$B = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad (28.2)$$

(magnetic field of a point charge with constant velocity)

$$\hat{i}\times\hat{j}=\hat{k}$$
$$\hat{j}\times\hat{k}=\hat{i}$$
$$\hat{k}\times\hat{i}=\hat{j}$$





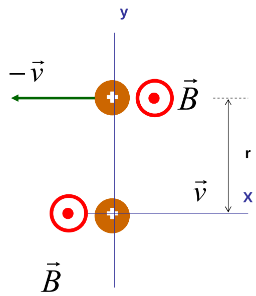
EXAMPLE 28.1 (modified)

EXECUTE

$$B = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$$

$$q = 1.6\times10^{-16}C$$

$$\mu_0 = 4\pi\times10^{-7}\frac{Tm}{A}$$



$$B = \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)}{4\pi}\frac{\left(1.6\times10^{-19}C\right)\left(10^{6}\frac{m}{s}\right)}{\left(10^{-6}m\right)^2} = 1.6\times10^{-8}T$$

Problem-Solving Strategy

Faraday's Law

IDENTIFY *the relevant concepts:* Faraday's law applies when there is a changing magnetic flux. To use the law, make sure you can identify an area through which there is a flux of magnetic field. This will usually be the area enclosed by a loop, usually made of a conducting material (though not always—see part (b) of Example 29.1). As always, identify the target variable(s).

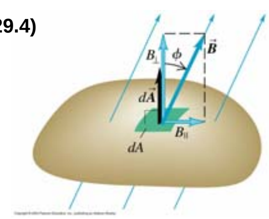
SET UP *the problem using the following steps:*

1. Faraday's law relates the induced emf to the rate of change of magnetic flux. To calculate this rate of change, you first have to understand what is making the flux change. Is the conductor moving? Is it changing orientation? Is the magnetic field changing? Remember that it's not the flux itself that counts, but its *rate of change*.
2. Choose a direction for the area vector $\vec{A}$ or $d\vec{A}$. The direction must always be perpendicular to the plane of the area. Note that you always have two choices of direction. For instance, if the plane of the area is horizontal, $\vec{A}$ could point straight up or straight down. It's like choosing which direction is the positive one in a problem involving motion in a straight line; it doesn't matter which direction you choose, just so you use it consistently throughout the problem.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law of induction)} \qquad (29.3)$$

$$\Phi_B = \vec{B}\bullet\vec{A} = BA\cos\phi \qquad \textbf{(29.2)}$$

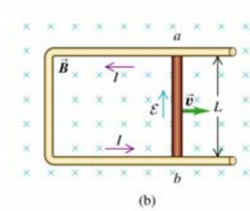
$$\varepsilon = -N\frac{d\Phi_B}{dt} \qquad \textbf{(29.4)}$$

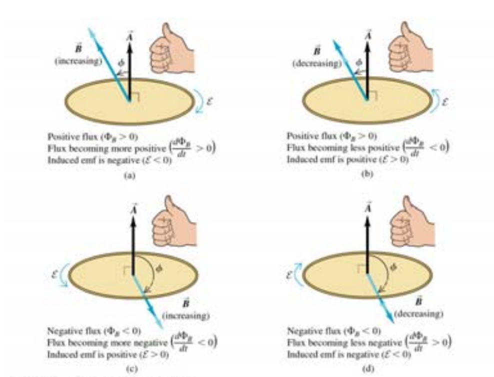


Motional Electromotive Force

$$\mathcal{E} = vBL \qquad (29.6)$$

(motional emf; length and velocity perpendicular to $\vec{B}$ uniform)





EXECUTE *the solution as follows:*

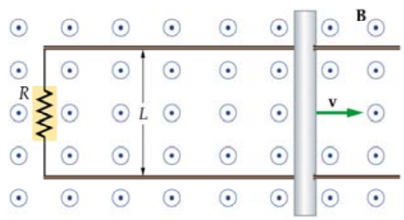
1. Calculate the magnetic flux using Eq. (29.2) if $\vec{B}$ is uniform over the area of the loop or Eq. (29.1) if it isn't uniform, being mindful of the direction you chose for the area vector.
2. Calculate the induced emf using Eq. (29.3) or (29.4). If your conductor has *N* turns in a coil, don't forget to multiply by *N*. Remember the sign rule for the positive direction of emf and use it consistently.
3. If the circuit resistance is known, you can calculate the magnitude of the induced current *I* using

$\mathcal{E} = IR$

EVALUATE *your answer:* Check your results for the proper units, and double-check that you have properly implemented the sign rules for calculating magnetic flux and induced emf.

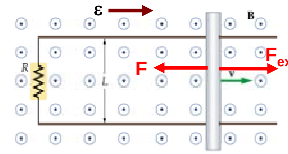
EXERCISE 29.20 (modified)

The figure shows a zero-resistance rod sliding on two zero-resistance rails.
In the circuit: R=20.0 Ω, L=0.8 m, and B=0.3 T.
(a) If the velocity of the bar is 10 m/s, what is the current in the circuit?
(b) Calculate induced electromotive force.
(c) Calculate an applied force that keeps the bar moving at constant velocity.



EXERCISE 29.20 (modified)

$$|\varepsilon| = IR = vBL$$



$\vec{F} = I\vec{l} \times \vec{B}$ (magnetic force on a straight wire segment) (27.19)

$$(a) I = \frac{vBL}{R} = \frac{(10m/s)(0.3T)(0.8m)}{20.0\Omega} = 0.012A$$

$$(b) \varepsilon = IR = (0.012A)(20.0\Omega) = 0.24V$$

$$(c) F_{ex} = IBL = (0.012A)(0.3T)(0.8m) = 2.88mN$$

Problem-Solving Strategy

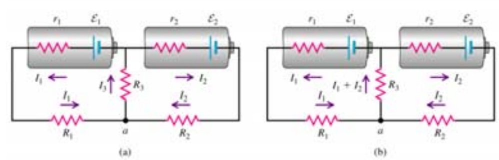
Kirchhoff's Rules

IDENTIFY *the relevant concepts:* Kirchhoff's rules are important tools for analyzing any circuit more complicated than a single loop.

SET UP *the problem using the following steps:*

1. Draw a *large* circuit diagram so you have plenty of room for labels. Label all quantities, known and unknown, including an assumed direction for each unknown current and emf. Often you will not know in advance the actual direction of an unknown current or emf, but this doesn't matter. If the actual direction of a particular quantity is opposite to your assumption, the result will come out with a negative sign. If you use Kirchhoff's rules correctly, they will give you the directions as well as the magnitudes of unknown currents and emfs.
2. When you label currents, it is usually best to use the junction rule immediately to express the currents in terms of as few quantities as possible. For example, Fig. 26.8a shows a circuit correctly labeled; Fig. 26.8b shows the same circuit, relabeled by applying the junction rule to point *a* to eliminate I_3.
3. Determine which quantities are the target variables.

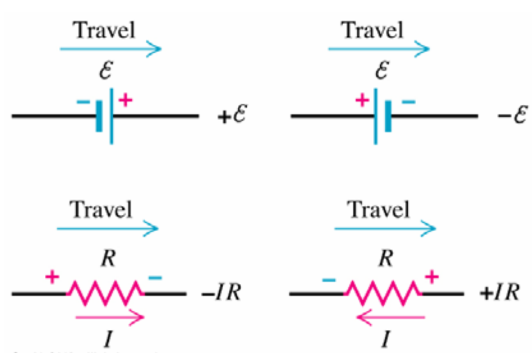
Junction Rule



26.8 Application of the junction rule to point *a* reduces the number of unknown currents from three to two.

EXECUTE *the solution as follows:*

1. Choose any closed loop in the network and designate a direction (clockwise or counterclockwise) to travel around the loop when applying the loop rule. The direction doesn't have to be the same as any assumed current direction.
2. Travel around the loop in the designated direction, adding potential differences as you cross them. Remember that a positive potential difference corresponds to an increase in potential and a negative potential difference corresponds to a decrease in potential. An emf is counted as positive when you traverse it from $(-)$ to $(+)$, and negative when you go from $(+)$ to $(-)$. An *IR* term is negative if you travel through the resistor in the same direction as the assumed current and positive if you pass it in the opposite direction. Figure 26.9 summarizes these sign conventions. In each part of the figure "travel" is the direction that we imagine going around a loop while using Kirchhoff's loop law, not necessarily the direction of current.
3. Equate the sum in Step 2 to zero.
4. If necessary, choose another loop to get a different relation among the unknowns, and continue until you have as many independent equations as unknowns or until every circuit element has been included in at least one of the chosen loops.



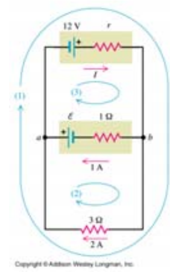
26.9 When using Kirchhoff's rules, follow these sign conventions as you travel around a circuit loop.

5. Solve the equations simultaneously to determine the unknowns. This step involves algebra, not physics, but it can be fairly complex. Be careful with algebraic manipulations; one sign error will prove fatal to the entire solution.

6. You can use this same bookkeeping system to find the potential V_{ab} of any point *a* with respect to any other point *b*. Start at *b* and add the potential changes you encounter in going from *b* to *a*, using the same sign rules as in Step 2. The algebraic sum of these changes is $V_{ab} = V_a - V_b$.

EVALUATE *your answer:* Check all the steps in your algebra. A useful strategy is to consider a loop other than the ones you used to solve the problem; if the sum of potential drops around this loop isn't zero, you made an error somewhere in your calculations. As always, ask yourself whether the answers make sense.

A multi-loop circuit



You can write one junction equation and two independent loop equations. Follow two out of three possible loops. Can calculate three unknowns.

Junction eq. - I + 1A + 2A = 0
I = 3A

Loop (1)eq. 12 V – (3 A)r – (2 A)(3 Ω) = 0
r = 2 Ω

Loop (2)eq. - ε + (1 A)(1Ω) – (2 A)(3 Ω) = 0
ε = - 5 V

EXERCISE 26.20 (modified)

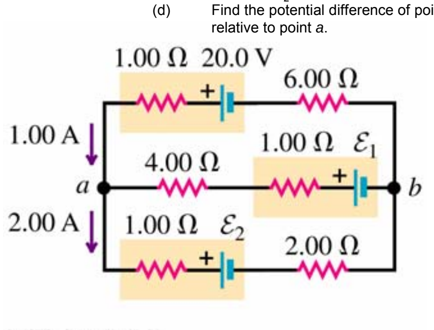
Consider the circuit.

(a) Find the current through the 4.00 Ω resistor.

(b) Find emf ε_1.

(c) Find emf ε_2.

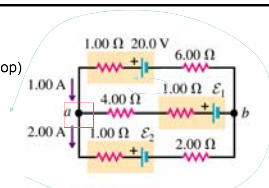
(d) Find the potential difference of point *b* relative to point *a*.



EXERCISE 26.20 (modified)

Identify: Kirchhoff's rules (junction and loop)
Sign conventions !

Set up: Choose the junction
Draw the loops



Execute: (a) $I_{ba} + 1.00A = 2.00A$

$I_{ba} = 1.00A$

(b) "small loop"

$$20.0\text{ V} - (1.00\text{ A})(6.00\,\Omega + 1.00\,\Omega) + (1.00\text{ A})(4.00\,\Omega + 1.00\,\Omega) - \varepsilon_1 = 0 \Rightarrow \varepsilon_1 = 18.0\text{ V}.$$

(c) "large" loop

$$20.0\text{ V} - (1.00\text{ A})(6.00\,\Omega + 1.00\,\Omega) - (2.00\text{ A})(1.00\,\Omega + 2.00\,\Omega) - \varepsilon_2 = 0 \Rightarrow \varepsilon_2 = 7.0\text{ V}.$$

(d)

$$V_{ab} = -(1.00\text{ A})(4.00\,\Omega + 1.00\,\Omega) + 18.0\text{ V} = +13.0\text{ V, so } V_{ba} = -13.0\text{ V}.$$

Evaluate